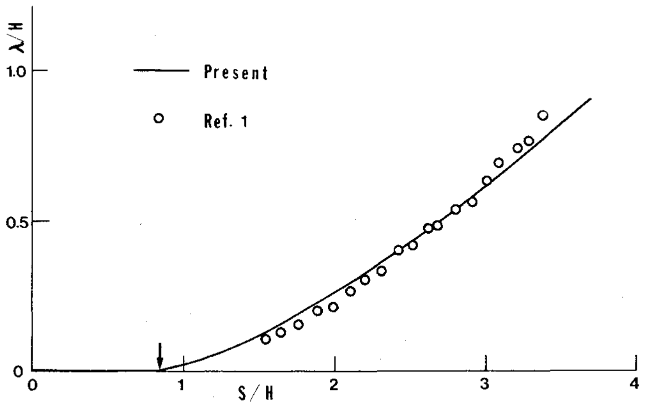

Fig. 3 Triple point trajectory for $\alpha = 90$ deg.

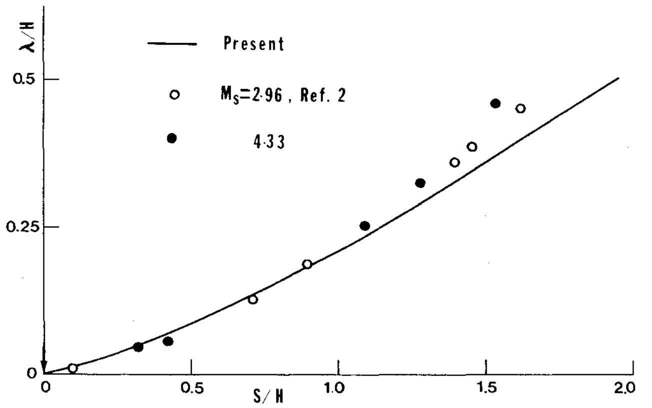

Fig. 4 Triple point trajectory for $\alpha = 30$ deg.

found between the present analysis and the experiment although the shock attenuation effect is ignored in this analysis. In Ref. 1, the spherical shock Mach number was 2.85 at $S/H = 1.36$, which attenuated to $M_s = 1.76$ at $S/H = 3.0$. However, a small discrepancy exists for $S/H > 3.0$. This is because in this analysis the deformation of a Mach stem and the shock attenuation are ignored, whereas in the experiment they become significant for larger S/H. An arrow in Fig. 3 indicates the initiation of Mach reflection. For a strong shock, the critical transition angle β_{crit} over a wedge is about 50 deg for $\gamma = 1.4$ from von Neumann's Detachment Criterion[5]; however, in the present Note this value is experimentally measured to be 44.0 ± 1.0 deg for $M_s > 2.9$ (for details see Ref. 2).

In Fig. 4, the analytical result, λ/H vs S/H for $\alpha = 30$ deg, is compared with the shock tube experiment[2] for $M_s = 2.96$ (data points are shown in open circles) and 4.33 (filled circles). Good agreement is found between the analysis and the experiment. In the shock tube experiment, the center of the spherical shock is slowly moving toward the apex of the cone, that is, the spherical shock is subjected to attenuation. Therefore, the discrepancy between the analysis and the experiment would result from this effect and the other factors previously mentioned.

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Similar Solutions of Unsteady, Laminar Boundary Layers on Swept Cylinders

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Nomenclature

$A_2, A_3, B_1, B_2 B_3$	= constants
C_1, C_2, C_3	= coefficients, Eq. (12)
d	= coefficient, Eq. (12)
F	= dimensionless streamfunction
F''_w	= wall shear-stress parameter in x direction
G	= velocity ratio v/V
G'_w	= wall shear-stress parameter in y direction
K	= constant
Q	= scaling function
t	= time
u,v,w	= velocity components in x, y, and z direction
U,V	= velocity components in x and y direction at the outer edge of the boundary layer
x,y,z	= coordinates
η	= similarity variable
ν	= kinematic viscosity
ψ	= streamfunction
ρ	= density

I. Introduction

SIMILAR solutions of the laminar boundary layer are still of importance as they represent test cases for approximate and numerical approaches. Similar solutions for steady and unsteady two-dimensional flows are well documented in the literature. Steady three-dimensional laminar boundary layers are given by Yohner and Hansen,[1] Hansen and Herzig,[2] and Christian.[3]

The problem of the unsteady laminar boundary layer in the stagnation region of an unswept cylinder was treated by Schuh.[4] Schuh discussed the governing equations, but did not present numerical results. Wuest[5] investigated the unsteady laminar boundary-layer flow in the stagnation region of a swept cylinder. In his work only the velocity component

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parallel to the cylinder axis was considered to be time dependent. In the present Note similar solutions for the stagnation region of swept cylinders are presented, where both the velocity components parallel and perpendicular to the cylinder axis are time dependent.

II. Governing Equations

The boundary-layer equations that describe the flow on swept cylinders are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2} \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}=\frac{\partial V}{\partial t}+\nu\frac{\partial^2 v}{\partial z^2} \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{3}$$

Using for the streamfunction $\psi(x, z, t)$

$$\psi=U(x,t)Q(t)F(\eta) \tag{4}$$

and for η

$$\eta=z/Q(t) \tag{5}$$

one gets

$$u=U(x,t)F'(\eta) \tag{6}$$

and

$$w=-F(\eta)Q(t)\frac{\partial U}{\partial x}(x,t) \tag{7}$$

where

$$F'=\mathrm{d}F/\mathrm{d}\eta \tag{8}$$

For the velocity component v the following expression is used:

$$v=V(t)G(\eta) \tag{9}$$

With Eqs. (6) and (7), Eqs. (1) and (2) result in

$$F'''-\frac{Q^2}{\nu}\frac{\partial U}{\partial x}(F'^2-FF''-1)+\frac{Q}{\nu}\frac{\mathrm{d}Q}{\mathrm{d}t}F''\eta-\frac{Q^2}{\nu U}\frac{\partial U}{\partial t}(F'-1)=0 \tag{10}$$

$$G''-\frac{Q^2}{\nu V}\frac{\mathrm{d}V}{\mathrm{d}t}(G-1)+\frac{Q^2}{\nu}\frac{\partial U}{\partial x}G'F+\frac{Q}{\nu}\frac{\mathrm{d}Q}{\mathrm{d}t}G'\eta=0 \tag{11}$$

For the case that Eqs. (10) and (11) are ordinary differential equations for the unknowns F and G, the expressions that depend on x and t must be constant. It follows that

$$C_1=\frac{Q^2}{\nu U}\frac{\partial U}{\partial t} \qquad C_3=\frac{Q}{\nu}\frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$C_2=\frac{Q^2}{\nu}\frac{\partial U}{\partial x} \qquad d=\frac{Q^2}{\nu V}\frac{\mathrm{d}V}{\mathrm{d}t} \tag{12}$$

Applying Eq. (12) leads to

$$F'''-C_2(F'^2-FF''-1)+C_3F''\eta-C_1(F'-1)=0 \tag{13}$$

and

$$G''-d(G-1)+C_2G'F+C_3G'\eta=0 \tag{14}$$

with the boundary conditions

$$\eta=0: \qquad F=F'=G=0$$

$$\eta\to\infty: \qquad F'=G=1 \tag{15}$$

III. Results

For the case $C_1=C_3=d=0$ and $C_2=1$, Eqs. (13) and (14) describe the steady boundary-layer flow along a swept cylinder, which was solved by Christian.[3] Wuest[5] has solved the problem $C_1=C_3=0$, $d\neq 0$ and $C_2=1$. In this case the velocity component U is not time dependent, only the velocity component parallel to the cylinder axis is a function of the time.

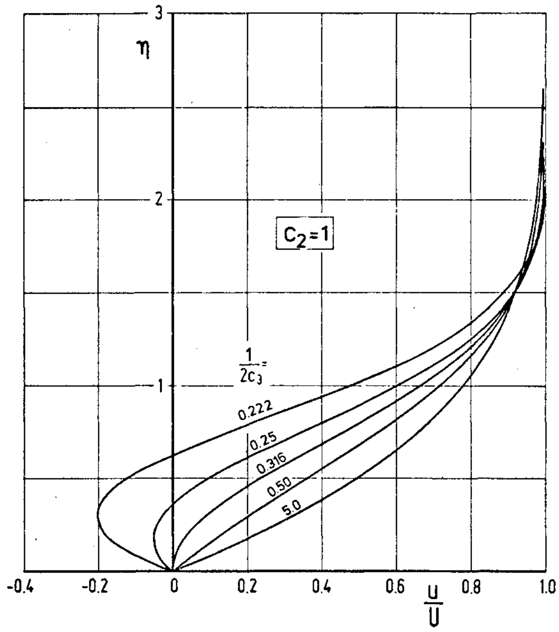

Fig. 1 Velocity profile u/U for $C_1=-2C_3$ with C_3 as a parameter.

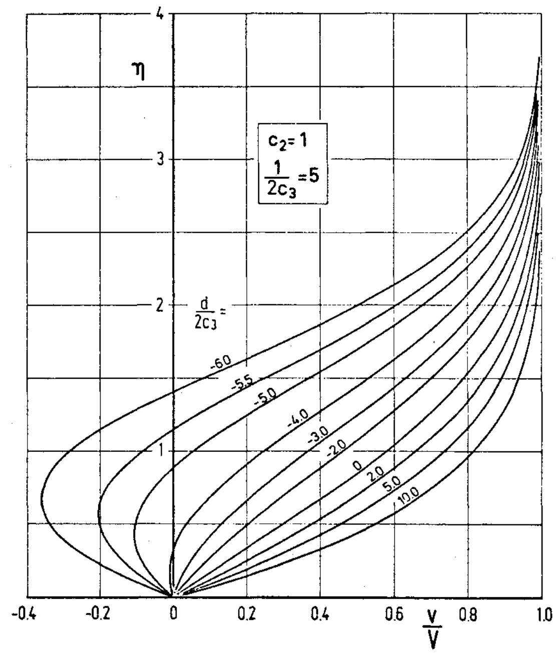

Fig. 2 Velocity profile v/V for $C_1=-2C_3$ and $C_3=0.1$ with d as a parameter.

Table 1 Wall shear stress components

$C_1 = -0.2$, $C_2 = 1.0$, $C_3 = 0.1$, $F''_w = 1.17462$		$C_1 = -3.164$, $C_2 = 1.0$, $C_3 = 1.582$, $F''_w = -0.02124$		$C_1 = -4.5$, $C_2 = 1.0$, $C_3 = 2.25$, $F''_w = -1.29653$	
d	G'_w	d	G'_w	d	G'_w
−1.2	−1.02754	−3.164	−2.08931	−3.6600	−1.86020
−1.1	−0.71128	−2.848	−1.25037	−3.375	−1.26442
−1.0	−0.47346	−2.531	−0.70937	−2.8125	−0.48602
−0.8	−0.12896	−2.215	−0.31875	−2.2500	0.02546
−0.6	0.11879	−1.900	−0.01534	−1.4085	0.56356
−0.4	0.31289	−1.266	0.44221	0.0	1.18158
0.0	0.61159	0.0	1.06631	3.3750	2.13709
0.4	0.84300	1.582	1.61008	6.7500	2.80921
1.0	1.12182	3.164	2.69766		
2.0	1.48803				

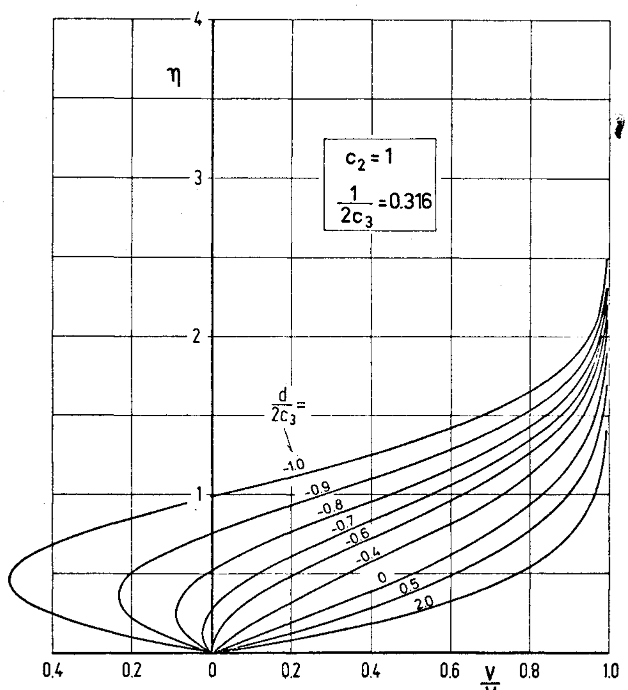

Fig. 3 Velocity profile v/V for $C_1 = -2C_3$ and $C_3 = 1.582$ with d as a parameter.

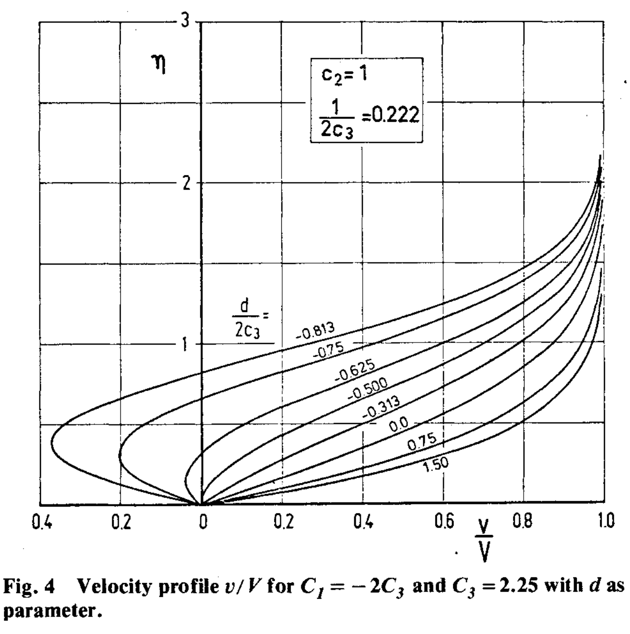

Fig. 4 Velocity profile v/V for $C_1 = -2C_3$ and $C_3 = 2.25$ with d as a parameter.

Taking all constants, Eq. (12), different from zero and using $C_2 = 1$ it follows that

$$C_3 > 0 \qquad C_1 = -2C_3 \qquad \psi = \frac{1}{2C_3}\frac{x}{t}(2\nu C_3 t)^{1/2} F(\eta)$$

$$\eta = z(2\nu C_3 t)^{-1/2} \qquad U = \frac{1}{2C_3}\frac{x}{t} \qquad V = B_1 t^{d/2C_3} \tag{16}$$

B_1 being an arbitrary positive constant. Equations (13) and (14) give

$$F''' - F'^2 + FF'' + 1 + C_3(2F' + F''\eta - 2) = 0$$

$$G'' - d(G-1) + G'F + C_3 G'\eta = 0 \tag{17}$$

Equation (16) describes a stagnation line flow where U is proportional to $1/t$ and V grows or decays with time for $d > 0$ or $d < 0$, respectively. For the pressure gradient in the x direction one gets

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = \frac{1}{2C_3}\frac{x}{t^2}\left(\frac{1}{2C_3} - 1\right) \tag{18}$$

Equation (18) shows that values $1/(2C_3) > 1$ result in accelerated and $1/(2C_3) < 1$ in decelerated flows in U.

Figure 1 gives the velocity profiles u/U with C_3 as a parameter. It appears that for values $C_3 > 1.6$ reversed flow in u/U occurs. In Figs. 2, 3, and 4 the velocity profiles v/V are given for d as a parameter with $C_3 = 0.1$, 1.582 and 2.25, respectively. For increasing values of C_3 the value of d, for which reversed flow in v/V is indicated, decreases; that means, for accelerated flows in U the deceleration in V can increase, before reversed flow in v/V is observed. In Table 1 the wall shear-stress components are given for different values of C_3 and d.

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